SECTION-A

1. $98 = 2 \times 7 \times 7$

$98 = 2 \times 7^2$ or $2^1 \times 7^2$

$$\begin{array}{r|l} 2 & 98 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

2. The graph intersects the x-axis at x = – 3 and x = – 1

∴ Zeroes of the polynomial are – 3 and – 1

3. For infinite many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{20} = \frac{5}{k} = \frac{-(k-5)}{-5}$$

Taking first two

$$\frac{k}{20} = \frac{5}{k}$$

$\Rightarrow k^2 = 100$

$\Rightarrow k = \pm 10$ ....(i)

Taking last two

$$\frac{5}{k} = \frac{(k-5)}{k}$$

$\Rightarrow 5 = k - 5$ $(\because k \neq 0)$

$\Rightarrow 10 = k$ ....(ii)

Common value of k = 10 (From (i) and (ii))

∴ k = 10 is the only solution

4. $\frac{1}{\sec\theta} = \cos\theta$

Maximum value of $\cos\theta$ is "One"

∴ Maximum value of $\frac{1}{\sec\theta}$ is "One"

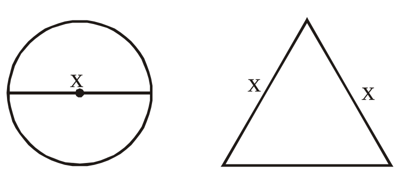
5. $\tan A = \frac{3}{4}$ ...(Given)

$\Rightarrow A + B = 90°$ $\Rightarrow A = 90° - B$

$\Rightarrow \tan(90° - B) = \frac{3}{4}$ ...[From (i)]

$\Rightarrow \cot B = \frac{3}{4}$ $[\therefore \tan(90° - \theta) = \cot\theta]$

6.



Let diameter of a circle = x units

then radius, $r = \frac{x}{2}$ = units

Let side of an eq. Δ = x unit

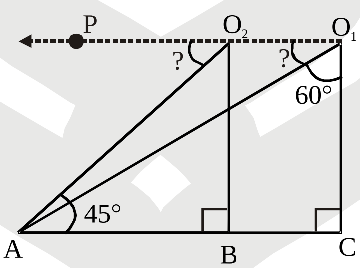
$$\frac{\text{ar. of a circle}}{\text{ar. of an eq. } \Delta} = \frac{\pi r^2}{\frac{\sqrt{3}}{4}(\text{side})^2}$$

$$= \frac{\pi\left(\frac{x}{2}\right)^2}{\frac{\sqrt{3}}{4}(x)^2} = \frac{\pi.\frac{x^2}{4}}{\sqrt{3}.\frac{x^2}{4}} = \frac{\pi}{\sqrt{3}}$$

$\therefore$ Required Ratio = $\pi : \sqrt{3}$

7. $\angle OQT = 90°$
$\angle TPO = 90°$ [Tangent is $\perp$ to teh radius through the point of contact]
In Quadrilateral POQT
$\angle POQ + \angle OQT + \angle QTP + \angle TPO = 360°$
(Sum of all angles of a Quad. is 360°)
$\Rightarrow$ $\angle POQ + 90° + 100° + 90° = 360°$
$\Rightarrow$ $\angle POQ + 280° = 360°$
$\therefore$ $\angle POQ = 360° - 280° = 80°$

8. Angle of depression from the observing position O_1 is $\angle PO_1A = 90° - 60° = 30°$



Angle of depression from the observing position O_2 is $\angle PO_2A = \angle O_2AB = 45°$
(Alternate Interior Angle)

9. S = {1, 2, 3, 4, 5, 6} i.e. n (S) = 6
Prime numbers are 2, 3, 5 i.e. n(E) = 3

$\therefore$ P (a prime number) = $\frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

10. The $\perp$ from the point of intersection of the two "Ogives" meets the x-axis at 4
$\therefore$ Median = 4 marks`

SECTION-B

11. Let a be the frist term and d be the common difference of an A.P.
$a_{10} = 47, a = 2$ (Given) ...(i)
$\Rightarrow a + 9d = 47$ [$\because a_n = a + (n-1)d$]
$\Rightarrow 47 = 2 + (10 - 1)d$
$\Rightarrow 47 = 2 + 9d$ $\Rightarrow 9d = 47 - 2 = 45$

$\therefore$ $d = \frac{45}{9} = 5$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{15} = \frac{15}{2}[2(2) + (15 - 1)(5)]$

$\Rightarrow \quad S_{15} = \frac{15}{2}$ [4 + (14_ (5)]

$\Rightarrow \quad S_{15} = \frac{15}{2}$ [4 + 70]

$\Rightarrow \quad S_{15} = \frac{15}{2}$ [74]

$\therefore \quad S_{15}$ = 15 (37) = 555

12. When we toss a coin, the outcomes head and tail are equally likely. Thus, the result of an individual coin toss is completely unpredictable.
Hence both the teams get equal chance to bat first so the given statement is justified.

13. $\frac{3}{x} + \frac{8}{y} = -1$ ...(i)

$\frac{1}{x} - \frac{2}{y} = 2$ ....(ii)

Multiplying (i) by 1 and (ii) by 4, we get

$\frac{3}{x} + \frac{8}{y} = -1$

$\frac{4}{x} - \frac{8}{y} = 8$ ......(by adding)

$\frac{7}{x} = 7$

$\Rightarrow$ 7x = 7 $\therefore$ x = 1

Putting the value of x in, (i), we get

$\frac{3}{1} + \frac{8}{y} = -1 \quad \Rightarrow \frac{8}{y} = -1 - 3 = -4$

$\Rightarrow$ –4y = 8 $\Rightarrow$ y = – 2

$\therefore$ x = 1, y = – 2

14. are (ΔABC) = 12 units^2

$\frac{1}{2}$ [4(2 – k) + (–3)(k – 1) + 0 (1 – 2)] = 12 units^2

[8 – 4k – 3k + 3] = ± 24

11 – 7k = ± 24

– 7k = ± 24 – 11

$k = \frac{\pm 24 - 11}{-7}$

$k = \frac{24 - 11}{-7}$ $\qquad k = \frac{-24 - 11}{-7}$

$k = \frac{+13}{-7}$ $\qquad k = \frac{-35}{-7}$

$\therefore k = \frac{-13}{7}$ $\qquad \therefore k = 5$

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15. Sum of zeroes, S = $2\sqrt{3}$

Proudct of zeroes, P = 2

Quadratic polynomial = $x^2 - Sx + P$

$= x^2 - 2\sqrt{3}x + 2$

SECTION-C

16. Let the original speed of the student = x km/h

Increased speed = (x + 1) = km/h

$\therefore \quad \frac{3}{x} - \frac{3}{x+1} = \frac{15}{60}$

$\Rightarrow \quad \frac{3x+3-3x}{x(x+1)} = \frac{1}{4} \qquad \left[\begin{array}{l} \because \text{ Time} = \frac{\text{distance}}{\text{speed}} \\ 15 \text{ mns} = \frac{15}{60}\text{hrs} \end{array}\right]$

$x(x + 1) = 12$

$x^2 + x - 12 = 0$

$x^2 + 4x - 3x - 12 = 0$

$x(x + 4) - 3(x + 4) = 0$

$(x + 4)(x - 3) = 0$

$x + 4 = 0$ or $x - 3 = 0$

$x = -4$ or $x = 3$

Rejecting x = –4, because speed cannot be -ve

$\therefore$ his original spped was 3km/h

17. Let us assume, to the contrary, that

$3 + 5\sqrt{2}$ is a rational number, say x

$\Rightarrow 5\sqrt{2} = x - 3 \quad \Rightarrow \quad \sqrt{2} = \frac{x-3}{5}$

Now x, 3 and 5 are all rational number

$\frac{x-3}{5}$ is also a rational number

$\Rightarrow \sqrt{2}$ is a rational number

$\Rightarrow \sqrt{2}$ is not a rational number

$\therefore$ our assumption is wrong

18. Here a = 2,. b = – (k – 2) = –k + 2 = 2 – k,

c = 1

D = 0 Equal roots ..... (Given)

$b^2 - 4ac = 0$

$\Rightarrow (2 - k)^2 - 4(2)(1) = 0$

$\Rightarrow 4 + k^2 - 4k - 8 = 0$

$\Rightarrow k^2 - 4k - 4 = 0$

Again here,

$A = 1, B = -4, C = -4$

$D = B^2 - 4AC$

$= (-4)^2 - 4\,(1)\,(-4)$

$= 16 + 16 = 32$

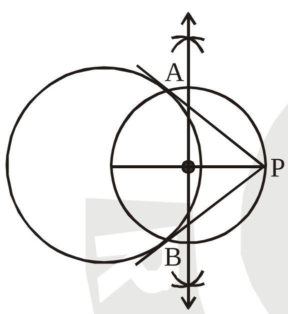
$\therefore \quad \sqrt{D} = \sqrt{16\times 2} = 4\sqrt{2}$

$$k = \frac{-B \pm \sqrt{D}}{2A} \quad \Rightarrow k = \frac{-(-4) \pm 4\sqrt{2}}{2(1)}$$

$$\Rightarrow \quad k = \frac{4 \pm 4\sqrt{2}}{2} \Rightarrow k = 2\left(\frac{2 \pm 2\sqrt{2}}{2}\right)$$

$\therefore \; k = 2 + 2\sqrt{2} \quad$ or $k = 2 - 2\sqrt{2}$

19. PA and PB are the requried tangents.



20.

$$\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$$

$$\Rightarrow \frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} - \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$$

L.H.S.

$$= \frac{\sin\theta\cot\theta - \sin\theta\operatorname{cosec} - \sin\theta\cot\theta - \sin\theta\operatorname{cosec}\theta}{(\cot\theta + \operatorname{cosec}\theta)\;(\cot\theta - \operatorname{cosec}\theta)}$$

$$= \frac{-2\sin\theta\,\operatorname{cosec}\theta}{\cot^2\theta - \operatorname{cosec}^2\theta}$$

$$= \frac{-2\left(\sin\theta \times \frac{1}{\sin\theta}\right)}{-1} \qquad [\because \cot^2\theta - \operatorname{cosec}^2\theta = -(\operatorname{cosec}^2\theta - \cot^2\theta = -1]$$

$= 2 = $ R.H.S

OR

$\sec 29^\circ = \sec\,(90^\circ - 61^\circ) = \operatorname{cosec}\,61^\circ$;

$\cot 17^\circ = \cot\,(90^\circ - 73^\circ) = \tan\,73^\circ$;

$\cot 8^\circ = \cot\,(90^\circ - 82^\circ) = \tan 82^\circ$;

$\sin^2 38^\circ = \sin^2\,(90^\circ - 52^\circ) = \cos^2 52$;

$\cot\,45^\circ = 1$

Using these values in the equation, we get

$$\frac{\sec 29^\circ}{\operatorname{cosec} 61^\circ} + 2\cot 8^\circ \cot 17^\circ \cot 45^\circ \cot 73^\circ \cot 82^\circ - 3(\sin^2 38^\circ + \sin^2 52^\circ)$$

$$= \frac{\operatorname{cosec}\ 61^\circ}{\operatorname{cosec}\ 61^\circ} + 2\tan 82^\circ \times \tan 73^\circ \times (1) \times \cot 73^\circ \times \cot 82^\circ - 3(\cos^2 52^\circ + \sin^2 52^\circ)$$

$$=1 + 2\tan 82^\circ \times \tan 73^\circ \times \frac{1}{\tan 73^\circ} \times \frac{1}{\tan 82^\circ} - 3(1)$$

($\because$ $\cos^2 A + \sin^2 A = 1$)

$= 1 + 2 - 3 = 0$

21. $\frac{XP}{XY} = \frac{XQ}{XZ} = \frac{3}{4}$ ...(Given)

$\angle X = \angle X$ (common)

$\Delta XPQ \sim \Delta XYZ$ (SAS similarty)

$$\Rightarrow \frac{ar\ \Delta XPQ}{ar\ \Delta XYZ} = \frac{XP^2}{XY^2}$$

$$\frac{ar\ \Delta XPQ}{ar\ \Delta XYZ} = \left(\frac{3}{4}\right)^2$$

[The ratio of the areas of two-Δs is equal to the ratio of the square of their correspoinding sides]

$$ar\ \Delta XPQ = \frac{9}{16} \times\ ar\ \Delta XYZ$$

$$= \frac{9}{16} \times 32 = 18\ cm^2$$

ar Quad. PYZQ = ar ΔXYZ – ar ΔXPQ

$\therefore$ ar Quad. PQZQ = $(32 - 18)\ cm^2 = 14\ cm^2$

22. Let AP : BP = k : 1

Coordinates of P = Co-ordinates of P

$$P\left(\frac{5k+3}{k+1}, \frac{3k-6}{k+1}\right)$$

This point lies on x-axis $\therefore \frac{3k-6}{k+1} = 0$

$\Rightarrow 3k - 6 = 0 \quad \Rightarrow 3k = 6$

$\Rightarrow k = \frac{6}{3} = 2$ Hence the ratio 2 : 1

Putting k = 2 in (i), we get

Point of intersection,

$$P = \left(\frac{5(2)+3}{2+1}, \frac{3(2)-6}{2+1}\right)$$

$$= \left(\frac{10+3}{3}, \frac{6-6}{3}\right) = \left(\frac{13}{3}, 0\right)$$

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23. Let P (x, y) be equidistant from the points A (2, 5) and B (–3, 7)

AP = BP ...(Given)

$\therefore$ $AP^2 = BP^2$ (Squaring both sides)

$(x-2)^2 + (y-5)^2 = (x+3)^2 + (y-7)^2$

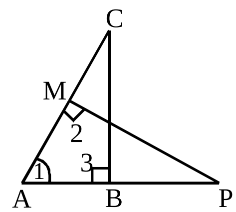
$\Rightarrow x^2 - 4x + 4 + y^2 - 10y + 25$

$= x^2 + 6x + 9 + y^2 - 14y + 49$

$\Rightarrow -4x - 10y - 6x + 14y = 9 + 49 - 4 - 25$

$\Rightarrow -10x + 4y = 29$

24. In ΔAMP and ΔABC



$\angle 1 = \angle 1$ (common)

$\angle 2 = \angle 3$ (each 90º)

$\Delta AMP \sim \Delta ABC$ (AA similarity)

$\therefore \dfrac{PA}{CA} = \dfrac{MP}{BC}$ (sides are proprotional)

$\therefore CA \times MP = PA \times BC$

25. $\theta = 360º - 120º = 240º$

$r = 3.5 \text{ cm} = \dfrac{35}{10} = \dfrac{7}{2}\text{cm}$

Length of OAPBO = length of arc BPA + OA + OB

$= \dfrac{\theta}{360}(2\pi r) + r + r$

$= \left(\dfrac{240}{360} \times 2 \times \dfrac{22}{7} \times \dfrac{7}{2}\right) + 2r$

$= \left(\dfrac{2}{3} \times 22\right) + \left(2 \times \dfrac{7}{2}\right)$

$= \dfrac{44}{3} + 7 = \dfrac{44+21}{3}$

$= \dfrac{65}{3} = 21\dfrac{2}{3}\text{cm}$

OR

Diameter AQ $= \dfrac{1}{4} \times 28 = 7\text{cm}$

$\Rightarrow r = \dfrac{7}{2}\text{cm}$

Diameter QB $= \dfrac{3}{4} \times 28 = 21\text{cm}$

$\Rightarrow R = \dfrac{21}{2}\text{cm}$

Area of shaded region $= \frac{1}{2}(\pi r^2 + \pi R^2)$

$= \frac{\pi}{2}(r^2 + R^2)$

$= \frac{1}{2}.\pi\left[\left(\frac{7}{2}\right)^2 + \left(\frac{21}{2}\right)^2\right]$

$= \frac{1}{2}\times\frac{22}{7}\times\left(\frac{49}{4}+\frac{441}{4}\right)$

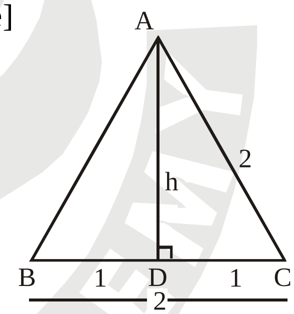
$= \frac{1}{2}\times\frac{22}{7}\times\left(\frac{49+441}{4}\right)$

$= \frac{11}{7}\times\frac{490}{4} = \frac{770}{4}$

$= 192.5 cm^2$

SECTION-D

26. $DC = \frac{BC}{2} = \frac{2}{2} = 1$ [In an eq. Δ, altitude bisects the base]



In rt. ΔADC

$AC^2 = AD^2 + DC^2$

(By the converse of given theorem)

$(2)^2 = h^2 + 1^2$

$h^2 = 4 - 1$

$h^2 = 3$

$\therefore\ h = \pm\sqrt{3}$ cm

(Height can not be -ve)

27. Let present age of sagar = x yrs.

and that of Tiru = y year

A.T.Q.

$x - 5 = 2(y - 5)$	$x + 10 = (y + 10) + 10$
$x - 5 = 2y - 10$	$x + 10 - y - 10 - 10$
$x - 2y + 5 = 0$	$x - y - 10 = 0$
$x = 2y - 5$	$x = y + 10$

x	5	15	25
y	5	10	15

x	15	20	25
y	5	10	15

Since the lines intersect at (25, 15)

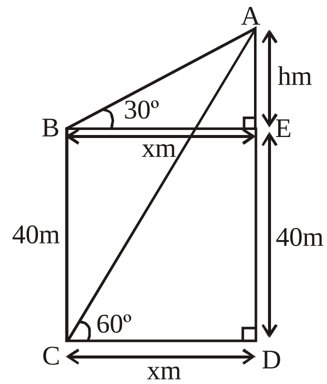
$\therefore$ Sagar's present age = 25yrs.

and Tiru's present age = 15 yrs.

From the graph it is clear that Sagar was 10 years old, when Tiru was born.

28. Let AE = h m and BE = CD = x m

$\therefore \quad \frac{x}{h} = \cot 30^\circ$



$\Rightarrow \frac{x}{h} = \sqrt{3}$

$\Rightarrow x = h\sqrt{3} \quad$ ....(i)

$\Rightarrow$ BE = CD = h $\sqrt{3}$ m

In rt. ΔADC, $\frac{AD}{CD} = \tan 60^\circ$

$\Rightarrow \frac{h+40}{x} = \sqrt{3}$

$\Rightarrow h + 40 = \sqrt{3}\,x$

$\Rightarrow h + 40 = \sqrt{3} \times h\sqrt{3}$

$\Rightarrow 40 = 3h - h \qquad 2h = 40$

$\Rightarrow h = 20m$

$\therefore$ Height of lighthouse = 20 + 40 = 60m

In rt. ΔADC, $\frac{AD}{AC} = \sin 60^\circ$

$\frac{60}{AC} = \frac{\sqrt{3}}{2} \qquad \Rightarrow \quad \sqrt{3}\ AC = 60 \times 2$

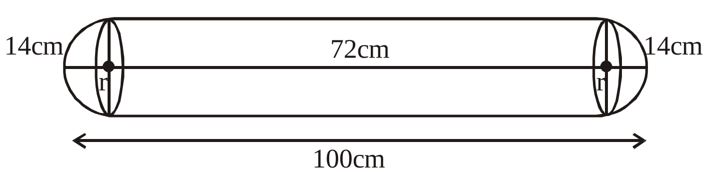
$AC = \frac{60 \times 2}{\sqrt{3}}$

$AC = 60 \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$AC = \frac{60 \times 2 \times \sqrt{3}}{3} = AC = 40\sqrt{3}m$

Hence the distance of the top of lighthouse from the foot of tower = $40\sqrt{3}$ m

29.



Radius of hemisphere, r = 14cm

length of cylindrical path (h) = [100 – 2 (14)] = 72 cm

Radius of cylindrical part

= Radius of hemispherical ends, r = 14cm

Total area to be polished

= 2 (C.S.A of hemispherical end) + C.S.A of cylinder

$= 2\,(2\pi r^2) + 2\pi rh = 2\pi r\,(2r + h)$

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$= 2 \times \frac{22}{7} \times 14\ (2 \times 14 + 72)$

$= 88\ (28 + 72) = 8800 cm^2$

Cost of polishing the surface

$= 8800 \times 0.05 = 440$

OR

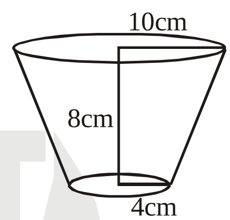
Height of container h = 8 cm

Radius of the base, R = 10 cm and r = 4cm

Slant height $l = \sqrt{h^2 + (R - r)^2}$

$= \sqrt{8^2 + (10-4)^2} = \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36} = \sqrt{100} = 10cm$



Volume of container

$= \frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)$

$= \frac{1}{3} \times 3.14 \times 8\ (100 + 16 + 40) cm^3$

$= \frac{1}{3} \times 3.14 \times 8\ (156)$

$= 1306.24\ cm^3$

$= \frac{1306.24}{1000}$ lit.

= 1.30624 lit. = 1.31 lit (approx)

$\therefore$ Quantity of oil = 1.31 lit

Cost of coil = Rs. (1.31 × 50) = Rs. 65.50

Surface area of the container (excluding the upper end)

= C.S are + ar of base

$= \pi l\ (R + r) = \pi r^2$

$= \pi[l\ (R + r) + r)^2$

$= 3.14 \times [10\ (10 + 4) + 16]$

$= 3.14 \times 156 = 489.84\ cm^2$

Cost of metal = Rs. $\left(489.84 \times \frac{5}{100}\right) = 24.492$

= 24.49 approx.

30.

Age	Number of people of f_i	Class mark (x_i)	$x_i f_i$
0-20	15	10	150
20-40	f_1	30	30 f_1
40-60	21	50	1050
60-80	f_2	70	70 f_2
80-100	17	90	1530
	100		2730 + 30f_1 + 70 f_2

$\sum x_i f_i = 2730 + 30f_1 + 70f_2$
$15 + f_1 + 21 + f_2 + 17 = 100$
$53 + f_1 + f_2 = 100$
$f_1 + f_2 = 100 - 53 = 47$
$f_2 = 47 - f_1$

Mean = $\frac{\sum x_i f_i}{\sum f_i}$

$\Rightarrow \frac{53}{1} = \frac{2730 + 30f_1 + 70f_2}{100}$
$\Rightarrow 2730 + 30f_1 + 70f_2 = 5300$
$\Rightarrow 30f_1 + 70f_2 = 5300 - 2730 = 2570$
$\Rightarrow 3f_1 + 7f_2 = 250$ (dividing by 10)
$\Rightarrow 3f_1 + 7(47 - f_1) = 257$ from (i)
$\Rightarrow 3f_1 + 329 - 7f_1 = 257$
$\Rightarrow -4f_1 = 257 - 329 = -72$

$\Rightarrow f_1 = \frac{-72}{-4} = 18$

Putting the value of f_1 in (i), we get
$f_2 = 47 - f_1 \Rightarrow f_2 = 47 - 18 = 29$
$\therefore f_1 = 18, f_2 = 29$

OR

Marks	Frequency	Cumulative Fequency c.f.
0-100	2	2
100-200	5	7
200-300	9	16
300-400	12	28
400-500	17	45
500-600	20	65
600-700	15	80
700-800	9	89
800-900	7	96
900-1000	4	100

$n = \sum f_i = 100 \quad \therefore \frac{n}{2} = 50$

Cumulative frequency just greater than $\frac{n}{2} = 50$ is 65 and corresponding class is 500 – 600

$\therefore$ median class is 500 – 600
$\therefore l = 500, \quad f = 20, \quad c.f. = 45, \quad h = 100$

$$\therefore \text{median} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h = 500 + \left(\frac{50-45}{20}\right) \times 100$$

$$\therefore \text{Median} = 500 + \left(\frac{5}{20} \times 100\right) = 500 + 25 = 525$$